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Path Related Hetro-Cordial Graphs

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Abstract – Let G = (V,E) be a graph with p vertices and q edges. A Hetro-Cordial labeling of a graph G with vertex set V is a bijection from V to {0, 1} such that each edge uv is assigned the label 0 if f(u) = f(v) or 1 if f(u) ≠ f(v) with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by atmost 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1. The graph that admits a Hetro-Cordial labeling is called a Hetro Cordial Graph (HeCG). In this paper, we proved that path related graphs Path P_n, Comp $P_n \Theta K_1$, Fan P_n+K_1, Double fan P_n+2K_1, Ladder P_n X K_2 are Hetro-Cordial Graphs.

Index Terms – Fan, Comp, Doublefan, Ladder, Hetro-Cordial Graph, Hetro-Cordial labeling, 2000 Mathematics Subject classification 05C78.

1. INTRODUCTION

A graph G is a finite nonempty set of objects called vertices together with a set of unordered pairs of distinct vertices of G which is called edges. Each pair e= {uv} of vertices in E is called edges or a line of G. In this paper, we proved that path related graphs Path P_n, Comp $P_n \Theta K_1$, Fan P_n+K_1, Doublefan P_n+2K_1, Ladder P_n X K_2 are Hetro-Cordial Graphs. For graph theory terminology, we follow [2].

2. PRELIMINARIES

Let G = (V,E) be a graph with p vertices and q edges. A Hetro-Cordial labeling of a Graph G with vertex set V is a bijection from V to {0, 1} such that each edge uv is assigned the label 0 if f(u) = f(v) or 1 if f(u) ≠ f(v) with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by atmost 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1.

The graph that admits a Hetro-Cordial labeling is called a Hetro-Cordial Graph (HCG). In this paper, we proved that path related graphs Path P_n, Comp $P_n \Theta K_1$, Fan P_n+K_1, Doublefan P_n+2K_1, Ladder P_n X K_2 are Hetro-Cordial Graphs.

Definition: 2.1

P_n is a path of length n-1.

Definition: 2.2

The join of G_1 and G_2 is the graph $G=G_1+G_2$ with vertex set $V=V_1 \cup V_2$ and edge set $E=E_1 \cup E_2 \cup$ {UV: $u \in V_1$, $v \in V_2$}. The graph P_n+K_1 is called a Fan and P_n+2K_1 is called the Doublefan.

Definition: 2.3

The product G_1 x G_2 of two graphs G_1 and G_2 is defined to be the graph whose vertex set is V_1 x V_2 and two vertices u= (u_1, u_2) and v= (v_1 ,v_2) in $V=V_1 x V_2$ are adjacent in G_1 x G_2 if either $u_1=v_1$ and u_2 is adjacent to v_2 or $u_2=v_2$ and u_1 is adjacent to v_1. $P_n X K_2$ is called a ladder.

Definition: 2.4

The corona $G_1 \Theta G_2$ of two graphs G_1 and G_2 is defined as the graph G obtained by taking one copy of G_1 (which has P_1 points) and P_1 copies of G_2 and joining the i^{th} point of G_1 to every point in the i^{th} copy of G_2. The graph $P_n \Theta K_1$ is called a comb.

3. MAIN RESULTS

Theorem: 3.1

Path P_n (n-odd) is Hetro-Cordial Graph.

Proof:

Let V (P_n) = {[u_i: $1 \le i \le n$]} and

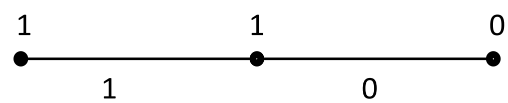
E (P_n) = {[($u_i u_{i+1}$): $1 \le i \le n-1$]}.

Define $f:$ V (P_n) → {0, 1}.

Case: 1

When n=3,

The labeling is,



Case: 2

When n>3,

The vertex labeling are,

$$f(u_i) = \begin{cases} 0 & i \equiv 0,3 \bmod 4 \\ 1 & i \equiv 1,2 \bmod 4 \end{cases} \quad 1 \le i \le n$$

The induced edge labeling are,

$$f^*[(u_i u_{i+1})] = \begin{cases} 1 & i \equiv 0 \bmod 2 \\ 0 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \le i \le n$$

Here, $v_f(1) = v_f(0) + 1$ for all n and

$e_f(1) = e_f(0)$ for all n.

Therefore, Path Pn satisfies the conditions $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$.

Hence, Path Pn (n-odd) is Hetro-Cordial Graph.

For example, Hetro-Cordial labeling of P5 is shown in the following fig 3.2

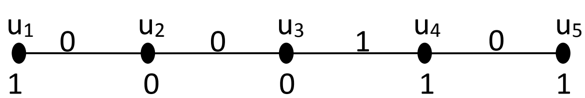


fig.3.2

Theorem: 3.3

Path P_n (n-even) is Hetro-Cordial Graph.

Proof:

Let $V(P_n) = \{[u_i: 1 \le i \le n]\}$ and

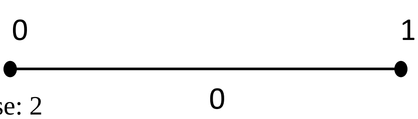
$E(P_n) = \{[(u_i u_{i+1}): 1 \le i \le n-1]\}$.

Define $f: V(P_n) \rightarrow \{0, 1\}$.

Case: 1

When n=2,

The labeling is,



Case: 2

When n >2,

The vertex labeling are ,

$$f(u_i) = \begin{cases} 0 & i \equiv 2,3 \bmod 4 \\ 1 & i \equiv 0,1 \bmod 4 \end{cases} \quad 1 \le i \le n$$

The induced edge labeling are,

$$f^*[(u_i u_{i+1})] = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \le i \le n-1$$

Here, $v_f(0) = v_f(1)$ for all n and

$e_f(1) = e_f(0) + 1$ for all n.

Therefore, Path P_n satisfies the conditions $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$.

Hence,Path P_n (n-even) is Hetro-Cordial Graph.

For example, Hetro-Cordial labeling of P_6 is shown in the following fig 3.4

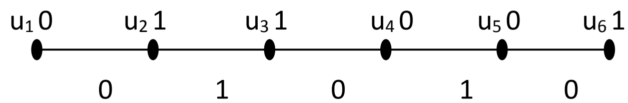


fig.3.4:P_6

Theorem: 3.5

Comp PnΘK1 is Hetro-Cordial Graph.

Proof:

Let $V(P_n \Theta K_1) = \{[u_i, v_i: 1 \le i \le n]\}$ and

$E(P_n \Theta K_1) = \{[(u_i u_{i+1}): 1 \le i \le n-1] \cup [(u_i v_i) : 1 \le i \le n]\}$.

Define $f: V(P_n \Theta K_1) \rightarrow \{0, 1\}$.

The vertex labeling are,

$f(u_i) = 0 \quad 1 \le i \le n$

$f(v_i) = 1 \quad 1 \le i \le n$

The induced edge labeling are,

$f^*[(u_i u_{i+1})] = 0 \quad 1 \le i \le n-1$

$f^*[(u_i v_i)] = 1 \quad 1 \le i \le n$

Here, $v_f(0) = v_f(1)$ for all n and

$e_f(0) = e_f(1) + 1$ for all n.

Therefore, comp PnΘK1 satisfies the conditions $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$.

Hence, Comp $P_n \Theta K_1$ is Hetro-Cordial.

For example, Hetro-Cordial labeling of P4ΘK1 and P3ΘK1 is shown in the following fig 3.6 and fig 3.7 respectively.

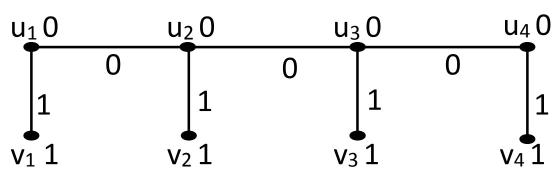


fig 3.6: $P_4 \Theta K_1$

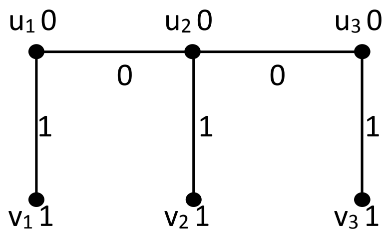


fig 3.7: $P_3 \odot K_1$

Theorem: 3.8

Fan P_n+K_1 (n-odd) is Hetro-Cordial Graph.

Proof:

Let $V(P_n+K_1) = \{[u, ui: 1 \leq i \leq n]\}$ and

$E(P_n+K_1) = \{[(uu_i): 1 \leq i \leq n] \cup [(u_iu_{i+1}): 1 \leq i \leq n-1]\}$.

Define $f : V(P_n+K_1) \rightarrow \{0, 1\}$.

The vertex labeling are,

$f(u) = 0$

$f(u_i) = \begin{cases} 0 & i \equiv 0,3 \bmod 4 \\ 1 & i \equiv 1,2 \bmod 4 \end{cases} \quad 1 \leq i \leq n$

The induced edge labeling are,

$f^*[(uu_i)] = \begin{cases} 0 & i \equiv 0,3 \bmod 4 \\ 1 & i \equiv 1,2 \bmod 4 \end{cases} \quad 1 \leq i \leq n$

$f^*[(u_iu_{i+1})] = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \quad 1 \leq i \leq n-1$

Here, $v_f(1) = v_f(0) + 1$ for all n and

$e_f(1) = e_f(0) + 1$ for all n.

Therefore, Fan P_n+K_1 (n-odd) satisfies the conditions $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence, Fan $Pn+K_1$ (n-odd) is Hetro-Cordial Graph.

For example, Hetro-Cordial labeling of P_5+K_1 is shown in the following fig 3.9

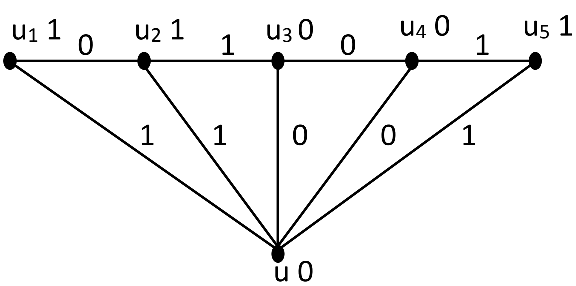


fig 3.9: P_5+K_1

Theorem: 3.10

Fan $Pn+K_1$ (n-even) is Hetro-Cordial Graph.

Proof:

Let $V(P_n+K_1) = \{[u, ui: 1 \leq i \leq n]\}$ and

$E(P_n+K_1) = \{[(uu_i): 1 \leq i \leq n] \cup [(u_iu_{i+1}): 1 \leq i \leq n-1]\}$.

Define $f : V(P_n+K_1) \rightarrow \{0, 1\}$.

The vertex labeling are,

$f(u) = 1$

$f(u_i) = \begin{cases} 0 & i \equiv 0,1 \bmod 4 \\ 1 & i \equiv 2,3 \bmod 4 \end{cases} \quad 1 \leq i \leq n$

The induced edge labeling are,

$f^*[(uu_i)] = \begin{cases} 0 & i \equiv 2,3 \bmod 4 \\ 1 & i \equiv 0,1 \bmod 4 \end{cases} \quad 1 \leq i \leq n$

$f^*[(u_iu_{i+1})] = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \leq i \leq n-1$

Here, $v_f(1) = v_f(0) + 1$ for all n and

$e_f(1) = e_f(0) + 1$ for all n.

Therefore, Fan P_n+K_1 (n-even) satisfies the conditions $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence, Fan $Pn+K_1$ (n-even) is Hetro-Cordial Graph.

For example, Hetro-Cordial labeling of P_6+K_1 is shown in the following fig 3.11

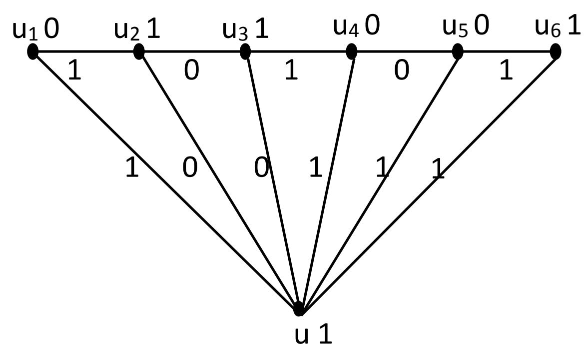


Fig 3.11: P_6+K_1

Theorem: 3.12

Ladder $Pn \times K_2$ (n-odd) is a Hetro-Cordial Graph.

Proof:

Let $V(P_n \times K_2) = \{[u_i, vi: 1 \leq i \leq n]\}$ and

E (P_n X K_2) = {[(u_iu_{i+1}) ∪ (v_iv_{i+1}): $1 \leq i \leq n-1$] ∪ [(u_iv_i): $1 \leq i \leq n$}.

Define $f : V(P_n \text{ X } K_2) \rightarrow \{0, 1\}$.

Case 1:

When n ≡ 1 (mod 4),

The vertex labeling are,

$f(u_i) = 0 \quad 1 \leq i \leq \frac{n+1}{2}$

$f(u_i) = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \quad \frac{n+3}{2} \leq i \leq n$

$f(v_i) = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \leq i \leq \frac{n-1}{2}$

$f(v_i) = 1 \quad \frac{n+1}{2} \leq i \leq n$

The induced edge labeling are,

$f^*[(u_iu_{i+1})] = 0 \quad 1 \leq i \leq \frac{n-1}{2}$

$f^*[(u_iu_{i+1})] = 1 \quad \frac{n+1}{2} \leq i \leq n-1$

$f^*[(v_iv_{i+1})] = 1 \quad 1 \leq i \leq \frac{n-1}{2}$

$f^*[(v_iv_{i+1})] = 0 \quad \frac{n+1}{2} \leq i \leq n-1$

$f^*[(u_iv_i)] = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \leq i \leq n$

Here, $v_f(0) = v_f(1)$ for all n and

$e_f(1) = e_f(0) + 1$ for all n.

Therefore, Ladder Pn X K_2 satisfies the conditions $| v_f(0) - v_f(1) | \leq 1$ and $| e_f(0) - e_f(1) | \leq 1$.

Case 2:

When n ≡ 3 (mod 4)

The vertex labeling are,

$f(u_i) = 0 \quad 1 \leq i \leq \frac{n+1}{2}$

$f(u_i) = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad \frac{n+3}{2} \leq i \leq n$

$f(v_i) = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \quad 1 \leq i \leq \frac{n-1}{2}$

$f(v_i) = 1 \quad \frac{n+1}{2} \leq i \leq n$

The induced edge labeling are,

$f^*[(u_iu_{i+1})] = 0 \quad 1 \leq i \leq \frac{n-1}{2}$

$f^*[(u_iu_{i+1})] = 1 \quad \frac{n+1}{2} \leq i \leq n-1$

$f^*[(v_iv_{i+1})] = 1 \quad 1 \leq i \leq \frac{n-1}{2}$

$f^*[(v_iv_{i+1})] = 0 \quad \frac{n+1}{2} \leq i \leq n-1$

$f^*[(u_iv_i)] = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \quad 1 \leq i \leq n$

Here, $v_f(0) = v_f(1)$ for all n and

$e_f(0) = e_f(1) + 1$ for all n.

Therefore,Ladder P_n X K_2 satisfies the conditions $| v_f(0) - v_f(1) | \leq 1$ and $| e_f(0) - e_f(1) | \leq 1$.

Hence,Ladder P_n X K_2 (n-odd) is a Hetro-Cordial Graph.

For example, Hetro-Cordial labeling of P3 X K_2 and P_5 X K_1 are shown in the following fig 3.13 and fig 3.14 respectively.

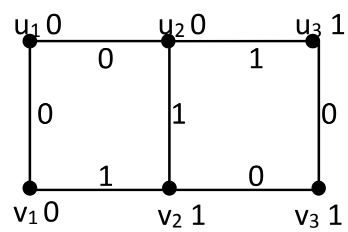


fig 3.13: P_3 X K_2

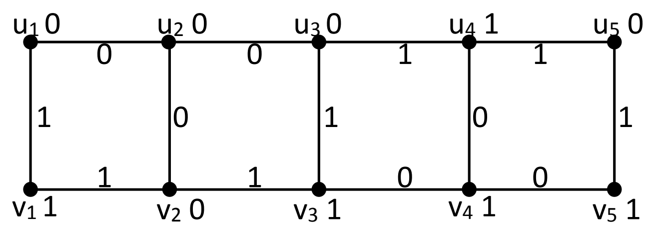


fig 3.14: P_5 X K_2

Theorem: 3.15

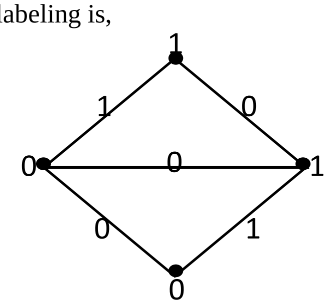
Doublefan P_n+2K_1 isHerto-Cordial Graph.

Proof:

Case: 1

When n=2,

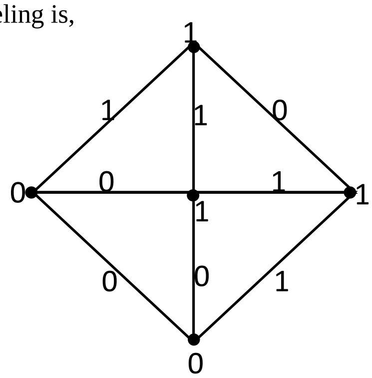
The labeling is,



Case: 2

When n=3,

The labeling is,



Case: 3

When n >3,

Let $V(P_n+2K_1) = \{[u, v, u_i: 1 \le i \le n]\}$ and

$E(P_n+2K_1) = \{[(uu_i): 1 \le i \le n] \cup [(vu_i): 1 \le i \le n] \cup [(u_iu_{i+1}): 1 \le i \le n-1]\}$.

Define $f : V(P_n+2K_1) \to \{0, 1\}$.

The vertex labeling are,

$$f(u_i) = \begin{cases} 0 & i \equiv 0,1 \bmod 4 \\ 1 & i \equiv 2,3 \bmod 4 \end{cases} \qquad 1 \le i \le n$$

$f(u) = 1$

$f(v) = 0$

The induced edge labeling are,

$$f^*[(u_iu_{i+1})] = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \qquad 1 \le i \le n-1$$

$$f^*[(uu_i)] = \begin{cases} 0 & i \equiv 2,3 \bmod 4 \\ 1 & i \equiv 0,1 \bmod 4 \end{cases} \qquad 1 \le i \le n$$

$$f^*[(vu_i)] = \begin{cases} 0 & i \equiv 0,1 \bmod 4 \\ 1 & i \equiv 2,3 \bmod 4 \end{cases} \qquad 1 \le i \le n$$

Here, $v_f(0) = v_f(1)$ for $n \equiv 0, 2 \bmod 4$,

$v_f(1) = v_f(0) + 1$ for $n \equiv 3 \bmod 4$,

$v_f(0) = v_f(1) + 1$ for $n \equiv 1 \bmod 4$,

$e_f(1) = e_f(0) + 1$ for $n \equiv 0, 2 \bmod 4$ and

$e_f(0) = e_f(1)$ for $n \equiv 1, 3 \bmod 4$.

Therefore, Doublefan $Pn+2K_1$ satisfies the conditions $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$.

Hence, Doublefan P_n+2K_1 is a Hetro-Cordial Graph.

For example, Hetro-Cordial labeling of P_4+2K_1 and P_5+2K_1 are shown in the following fig3.16 and fig 3.17 respectively.

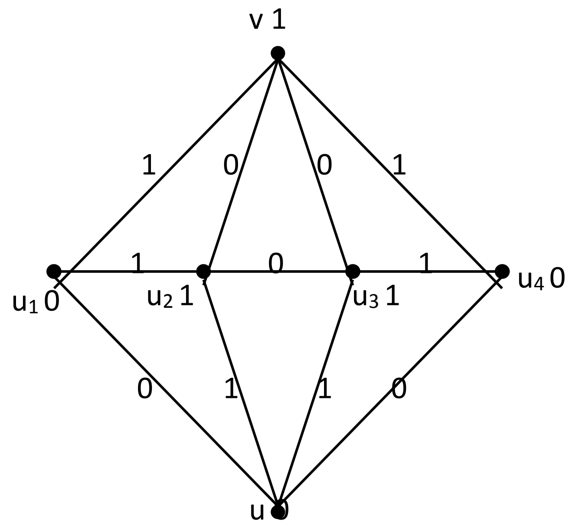


Fig 3.16: P_4+2K_1

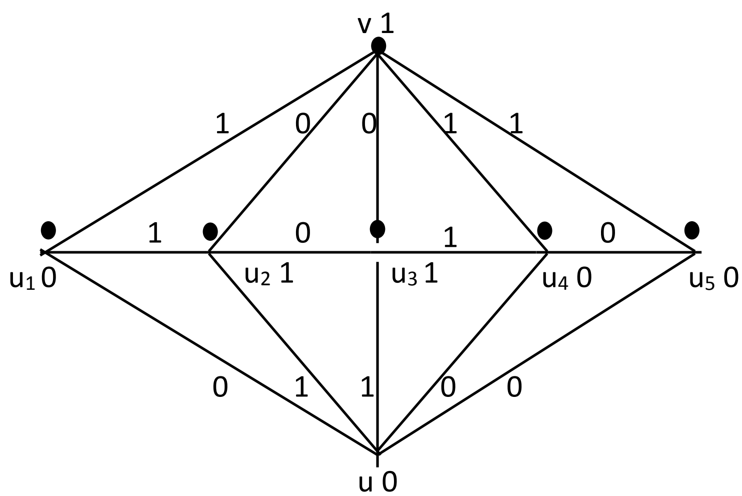


fig.3.17: P_5+2K_1

4. CONCLUSION

Hetro cordial is nothing but the principle is just a reverse of homo cordial. As homo cordial hetro cordial find its own applications.

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